AIAA JOURNAL
Vol. 31, No. 10, October 1993

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Transition Correlation in Subsonic Flow over a Flat Plate

J. A. Masad* and M. R. Malik†
High Technology Corporation, Hampton, Virginia 23666

THE need to predict transition location in flow over aerodynamic surfaces under different flow conditions is becoming increasingly important. The location of transition on an aerodynamic surface has a significant effect on the amount of drag force. Therefore, an estimate of the transition location on aerodynamic surfaces is critical for the design of such surfaces. Although the transition process in a two-dimensional flow over a flat plate is different in many aspects than that for a three-dimensional flow (such as a flow over a swept wing), many theoretical and experimental studies have been performed to understand the receptivity, stability, and transition mechanism for two-dimensional flow over a flat plate.

The most common approach for estimating the location of transition onset in flow over a flat plate is the theoretical-empirical e^N approach proposed by Smith and Gamberoni[1] (see also Jaffe et al.[2]). In this approach, transition is assumed to occur when the integrated disturbance growth reaches a certain value. Although the disturbance growth rates are computed with linear stability theory, the integration of the growth rates (called the N factor) is empirically correlated with transition. This correlation is based on available experimental data. For low-speed flow over a flat plate, transition occurs when the N factor reaches a value close to 9. Note that the N factor approach does not account for nonlinear effects, secondary parametric-excitation instabilities, spectral structure, level of initial and freestream disturbances, or any wave-wave interactions.

In this Note, we consider a two-dimensional subsonic flow over a flat plate with a freestream Mach number M_∞ up to 0.8. The flow can experience continuous uniform suction (with constant suction velocity) through the wall, and the wall can be heated or cooled continuously with a fixed wall temperature. For a specific combination of freestream Mach number, suction velocity, and level of heat transfer, the mean-flow problem is solved and linear stability calculations are performed to compute the location on the flat plate where the N factor reaches 9. These calculations are repeated for several combinations of flow parameters, and the theoretically predicted transition location is presented in the form of a correlation that can account for the effect of wall suction, heat transfer, and Mach number.

The continuous, uniform nondimensional suction velocity is denoted by v_w such that $v_w = v_w^*/U_\infty^*$, where v_w^* is the dimensional suction velocity and U_∞^* is the dimensional freestream velocity. The actual nondimensional wall temperature is denoted by T_w, and the adiabatic nondimensional wall temperature is denoted by T_{ad}. Both T_w and T_{ad} are made nondimensional with respect to the dimensional freestream temperature T_∞^* such that $T_w = T_w^*/T_\infty^*$ and $T_{ad} = T_{ad}^*/T_\infty^*$. In specifying the level of heat transfer, the ratio T_w/T_{ad} of the wall temperature to the adiabatic wall temperature is specified. When T_w/T_{ad} is less than unity, the plate is cooled; when it is larger than unity, the plate is heated. When $T_w/T_{ad} = 1$, the plate is adiabatic. An approximate, but rather accurate, formula for the variation of T_{ad} with M_∞ is given by (see Schlichting[3])

$$T_{ad} = 1 + (\gamma - 1)/2\sqrt{Pr}M_\infty^2 \tag{1}$$

where Pr is the Prandtl number and γ is the ratio of specific heats that is assumed to be constant and equal to 1.4. When Eq. (1) is used to calculate T_{ad}, the error increases as M_∞ increases. For $Pr = 0.72$, the percentage error with Eq. (1) varies from 0% at $M_\infty = 0$ to 0.016% at $M_\infty = 1$. The application of suction has virtually no effect on the accuracy of Eq. (1). In the mean flow and the stability calculations, the specific heat at constant pressure C_p^* is constant; the variation of dynamic viscosity with temperature is given by the Sutherland formula; and the freestream Prandtl number $Pr = \mu_\infty^* c_p^*/\kappa_\infty^*$ is constant and equal to 0.72, where μ_∞^* is the freestream dimensional dynamic viscosity and κ_∞^* is the freestream dimensional thermal conductivity. The freestream temperature is fixed at 300 K. The dimensional freestream kinematic viscosity is denoted by ν_∞^* such that $\nu_\infty^* = \mu_\infty^*/\rho_\infty^*$, where ρ_∞^* is the dimensional freestream density. The nondimensional frequency F is given by $2\pi f^* \nu_\infty^*/U_\infty^{*2}$, where f^* is the dimensional frequency (in Hz) of the disturbance. Because F is proportional to f^*, it remains constant following the same wave as it is convected downstream. The local Reynolds number Re_x is given by $Re_x = U_\infty^* x^*/\nu_\infty^*$, where x^* is the dimensional distance from the leading edge to the location where the stability calculations are performed. The freestream Reynolds number $Re = U_\infty^* L^*/\nu_\infty^*$ is related to Re_x through $Re_x = xRe$, where $x = x^*/L^*$ and L^* is a constant length scale. The disturbances considered in this work are two dimensional because, as shown by Mack,[4] two-dimensional waves are the most amplified waves in subsonic flow with a freestream Mach number up to approximately 0.8. An assumption is made that suction and wall heat transfer do not effect this behavior. The nonsimilar mean flow and the compressible linear stability equations are available in many references (e.g., Mack[4]).

The combination of the freestream Mach number M_∞, the uniform suction velocity v_w, and the ratio of actual wall temperature to adiabatic wall temperature T_w/T_{ad} considered in this study is within certain bounds. The freestream Mach number M_∞ varies from 0 to 0.8, v_w varies from 0 to -2×10^{-5}, and T_w/T_{ad} varies from 0.95 to 1.05. Over 80 combinations were chosen for this study. For each combination, the mean-flow problem was solved and the stability calculations were then performed at a fixed value of F to determine the location Re_x where the N factor reached a value of 9. Then F was varied, and the new location at which the N factor reached the value 9 was determined, and so on. The value of F that resulted in the smallest value of Re_x at $N = 9$ was considered to be the frequency responsible for transition; and the corresponding location was considered as the predicted transition location, and its Reynolds number was denoted by $(Re_x)_{N=9}$. The frequency F was varied in steps of 1×10^{-6}.

Received Feb. 11, 1993; accepted for publication April 20, 1993.

*Research Scientist, P.O. Box 7262. Member AIAA.
†President, P.O. Box 7262. Senior Member AIAA.

The frequency responsible for transition was found to decrease as the predicted transition location moved downstream (Fig. 1). The downstream movement of the predicted transition location results from an increase in M_∞, an increase in the suction velocity, or a decrease in T_w/T_{ad} to cool the surface. At $M_\infty = 0$, $v_w = 0$, and $T_w/T_{ad} = 1$, the frequency responsible for transition is equal to 26×10^{-6} (the corresponding value at $M_\infty = 0.8$ is 15×10^{-6}). An increase in cooling, suction, or M_∞ can take the frequency responsible for transition in the considered combinations down to $F = 10 \times 10^{-6}$, whereas a decrease in cooling (or an increase in heating) in the considered combinations can take the frequency responsible for transition up to 30×10^{-6}. The frequency F correlates with $(Re_x)_{N=9}$ as

$$(Re_x)_{N=9} = 254\Gamma \tag{2}$$

where

$$\Gamma = 1700 + \frac{10^6}{(F \times 10^6)^{1.39}} \tag{3}$$

A plot of the variation of the calculated and correlated $(Re_x)_{N=9}$ with F is shown in Fig. 1. In general, the agreement is good. To correlate $(Re_x)_{N=9}$ with M_∞, v_w, and T_w/T_{ad}, we considered a correlation of the form

$$(Re_x)_{N=9} = a^2\Lambda^2 \tag{4a}$$

$$\Lambda = \Lambda_1\Lambda_2\Lambda_3 \tag{4b}$$

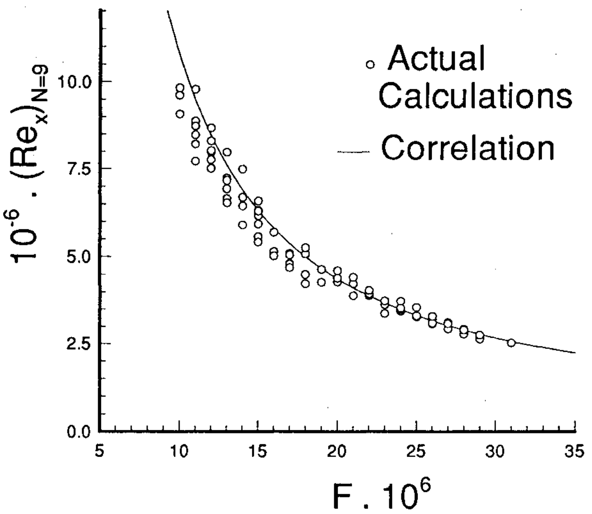

Fig. 1 Variation of predicted transition location with frequency predicted to be responsible for transition.

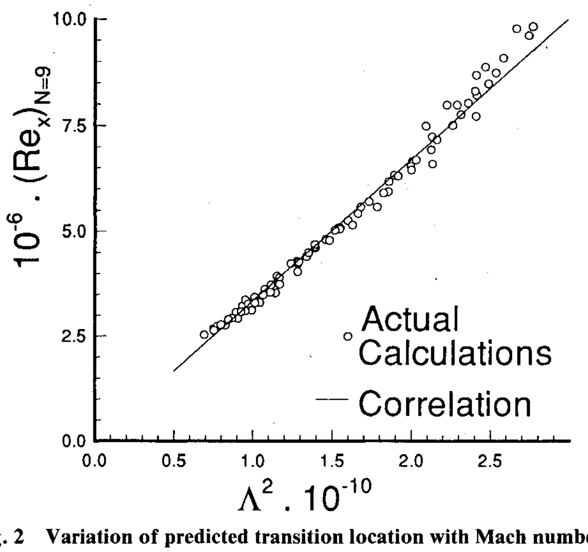

Fig. 2 Variation of predicted transition location with Mach number, suction, and heat-transfer parameter.

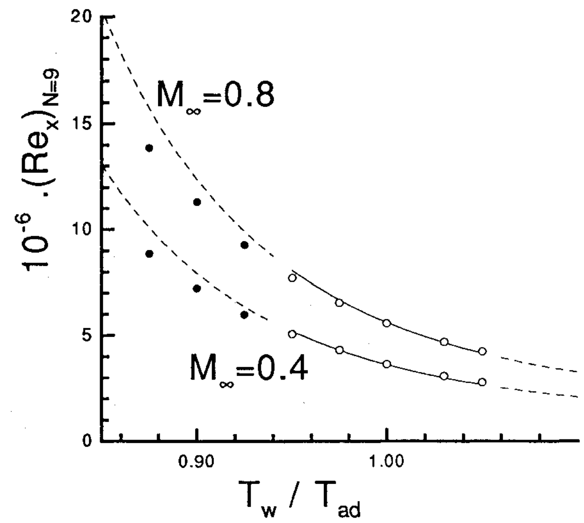

Fig. 3 Variation of predicted transition location with T_w/T_{ad} at $v_w = 0$.

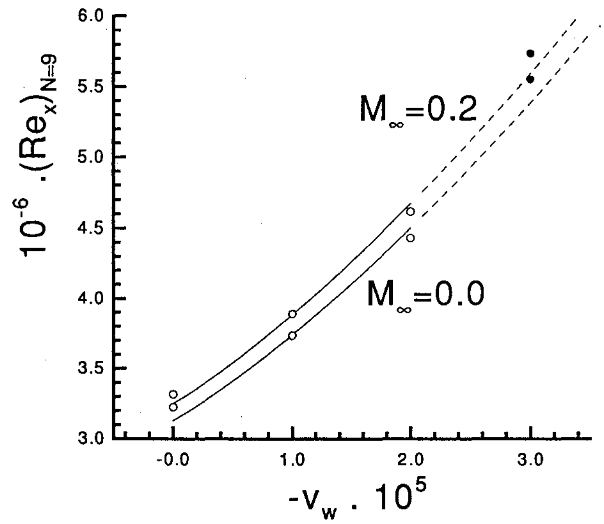

Fig. 4 Variation of predicted transition location with v_w at $T_w/T_{ad} = 1$.

where

$$\Lambda_1 = \alpha_1 + M_\infty^{\alpha 2} + \alpha_3 M_\infty^{\alpha 4} \tag{5}$$

$$\Lambda_2 = b_1 + 10^8 |v_w|^{b_2} + b_3 10^8 |v_w|^{b_4} \tag{6}$$

$$\Lambda_3 = c_1 + (T_{ad}/T_w)^{c_2} + c_3(T_{ad}/T_w)^{c_4} \tag{7}$$

This form is consistent with the stabilizing effects of increasing the Mach number, suction, and cooling level. The constants α, α_i, b_i, and c_i were obtained by minimizing both the average percentage difference and the maximum percentage difference between the directly computed and correlated $(Re_x)_{N=9}$. These constants are equal to

$$\alpha = 0.0183, \quad \alpha_1 = 4.1, \quad \alpha_2 = 3.5, \quad \alpha_3 = 1.41, \quad \alpha_4 = 1.83$$

$$b_1 = 1310, \quad b_2 = 1.28, \quad b_3 = 0.082, \quad b_4 = 1.0$$

$$c_1 = 7.9, \quad c_2 = 5.8, \quad c_3 = 9.1, \quad c_4 = 5.9$$

The variation of the directly computed $(Re_x)_{N=9}$ and the correlated $(Re_x)_{N=9}$ with Λ^2 is shown in Fig. 2. The agreement is very good. The average percentage difference between the directly computed $(Re_x)_{N=9}$ and the correlated $(Re_x)_{N=9}$ is 2.5%, and the maximum percentage difference is 8.8%. In Fig. 2a, $(Re_x)_{N=9}$ varies from about 2.5 million to 9.5 million.

The correlation of $(Re_x)_{N=9}$ with T_w/T_{ad} at two values of the freestream Mach number and $v_w = 0$ is shown in Fig. 3 in addition to 16 points from the actual calculations. The dashed lines and filled circles in the figure indicate conditions outside the range of data used to obtain the correlation. However, the agreement between the correlation and actual calculations at these conditions is very good. In Fig. 4, we show the correlation of $(Re_x)_{N=9}$ with v_w at two values of the freestream Mach number and $T_w/T_{ad} = 1$ with 8 points from the actual calculations. The agreement between the actual calculation and the correlation at $v_w = -3 \times 10^{-6}$ (which is outside the range of data used to obtain the correlation) is satisfactory.

Mack[5] correlated low-speed experimental transition data with wind-tunnel turbulence level and suggested the N factor correlation

$$N_T = -8.43 - 2.4 \ln Tu \tag{8}$$

where Tu is the turbulence level.

Bushnell et al.[6] reported some experimental evidence that suggests the previous correlation (8) can be used even at higher subsonic Mach numbers. Hence, Eqs. (4) and (8) together can be used to estimate the effect of suction, wall heat transfer, and freestream turbulence on transition location, whereby, with $N = 9$ as the reference case, Eq. (8) can provide information about the forward movement of transition caused by freestream turbulence.

Acknowledgment

This research is supported by the Theoretical Flow Physics Branch, Fluids Mechanics Division, NASA Langley Research Center, Hampton, Virginia, under Contract NAS1-19299.

References

[1]Smith, A. M. O., and Gamberoni, N., *Transition, Pressure Gradient, and Stability Theory,* Douglas Aircraft Co., Inc., Rept. ES 26388, El Segundo, CA, Aug. 1956.

[2]Jaffe, N. A., Okamura, T. T., Smith, A. M. O., "Determination of Spatial Amplification Factors and their Application to Predicting Transition," *AIAA Journal,* Vol. 8, No. 2, 1970, pp. 301–308.

[3]Schlichting, H., *Boundary-Layer Theory,* 7th ed., McGraw-Hill, New York, 1979, p. 334.

[4]Mack, L. M., *Boundary-Layer Stability Theory,* Jet Propulsion Lab. Document 900-277, Rev. A, California Inst. of Technology, Pasadena, CA, May 1969.

[5]Mack, L. M., *Transition Prediction and Linear Stability Theory,* AGARD CP-224, May 1977.

[6]Bushnell, D. M., Malik, M. R., and Harvey, W. D., *Transition Prediction in External Flows Via Linear Stability Theory,* presented at the International Union of Theoretical and Applied Mechanics Symposium Transsonicum III, Göttingen, West Germany, May 1988.

Application of a Generalized Minimal Residual Method to Two-Dimensional Unsteady Flows

R. Hixon* and L. N. Sankar†

Georgia Institute of Technology, Atlanta, Georgia 30332

Introduction

DURING the past two decades, there has been significant progress in the field of numerical simulation of unsteady compressible flowfields (e.g., transonic full potential equation-based methods, unsteady Euler solvers, and unsteady Navier-Stokes solvers). Despite the advances in algorithms and computer hardware, the numerical simulation of unsteady viscous flows remains a computationally intensive problem, even in two dimensions. Because of the excessive CPU requirements of the existing unsteady viscous flow solvers, dramatic improvements in algorithms and/or computer architecture are necessary before unsteady viscous flow analyses become practical day-to-day tools.

One scheme that has been of recent interest is the generalized minimal residual (GMRES) method originally proposed by Saad and Schultz.[1] This procedure uses a conjugate-gradient-like method to accelerate the convergence of existing solvers and was used by Wigton et al.[2] to accelerate computational fluid dynamics codes to steady solutions. In this work, the GMRES scheme has been considered as a candidate for acceleration of a Newton-iterative time-marching scheme for unsteady two-dimensional compressible viscous flow calculations. GMRES has provided significant reductions in the computer time requirements over the existing class of explicit and implicit time marching schemes.

Numerical Formulation

The starting point for the GMRES method is an existing flow solver, which is used as a preconditioner for the flow equations. A Newton iterative time marching scheme that solves the two-dimensional compressible Navier-Stokes equations in a curvilinear body-fitted coordinate system has been used as a preconditioner.[3] The solver employs approximate factorization of the spatial inviscid flux terms, and uses the Baldwin-Lomax turbulence model.

Given a known flowfield at the nth time level, the Newton iterative solver takes an initial guess for the flowfield at the next time level $(n+1)$, and iterates in order to improve the answer at the new time level. The advantage of the Newton iteration in this application is that the errors associated with the approximate factorization can be reduced or removed.

The Newton iterative solver may be written as

$$[A]^{n+1,k}\{\Delta q\}^{n+1,k+1} = \{R\}^{n+1,k} \tag{1}$$

where $n+1$ and k are the time level and iteration level, respectively, A is the preconditioning matrix, Δq is the correction vector for the flow quantities, and R is the residual of the discretized equation at the last iteration level.

Generalized Minimal Residual Formulation

The Newton iterative solver may also be written as

$$M(q^{n+1,k}) = \{\Delta q\}^{n+1,k+1} = [A]^{-1}\{R\}^{n+1,k} = 0 \tag{2}$$

When Eq. (2) is satisfied, the flowfield is converged at the new time level $n+1$.

The GMRES formulation that is used in this investigation is documented in Refs. 1 and 2. The GMRES solver uses the Newton solver as a function evaluator and computes the vector of flow properties $q^{n+1,k}$ that will satisfy Eq. (2).

It should be noted here that the GMRES routine uses the Newton iterative solver as a "black box" to determine the effect of changing the input flow properties on the correction vector Δq. Because of this, the GMRES solver is very portable and can be easily implemented in a wide variety of codes regardless of the original code's solution procedure [as long as a residual $M(q)$ can be defined]. This is a major advantage of the GMRES acceleration method over schemes that are tied closely to the details of the algorithm (e.g., multigrid methods).

The GMRES solver starts by assuming that the Δq required to set the residual given by Eq. (2) to zero lies in a vector space defined by a set of orthogonal direction vectors. In a two-dimensional flow problem, there are a total of 4**i*max**k*max possible direction vectors. The GMRES method uses the original code as a preconditioner to the problem to define J (usually < 20) orthogonal unit direction vectors which hopefully contain a majority of the error components. After computing the slope of the residual in

Presented as Paper 92-0422 at the AIAA 30th Aerospace Sciences Meeting, Reno, NV, Jan. 6–9, 1992; received April 19, 1992; revision received Oct. 15, 1992; accepted for publication March 11, 1993.

*Graduate Research Assistant, School of Aerospace Engineering. Member AIAA.

†Professor, School of Aerospace Engineering. Senior Member AIAA.